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MAGNETIC CEPHEIDS

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ABSTRACT

The role of a magnetic field in the pulsational behavior of classical Cepheids has been studied by computing linearized models of pulsating stellar envelopes pervaded by a well-tangled magnetic field. It is found that the "pulsational" masses of Cepheids that are implied by the theoretical values of the quantities Q_0, P_1/P_0, and (possibly) P_2/P_0 can be brought into line with the large "evolutionary" masses, if the pressure due to the postulated magnetic field is assumed to be comparable to the thermodynamic pressure everywhere in the pulsating layers. The field strengths that are required—several 10^2 gauss at the surface and several 10^4 gauss at the base—are quite reasonable from the point of view of the observed magnetic fields in Cepheids. The influence of the magnetic field on the location of the blue edge of the instability strip in the H-R diagram is estimated to be very slight.

Subject headings: stars: Cepheids — stars: interiors — stars: magnetic — stars: pulsation

I. INTRODUCTION

The discovery of magnetic fields in a few classical Cepheids (Babcock 1958; Kraft 1967; Weiss and Wood 1975; Wood, Weiss, and Jenkner 1977) raises the interesting question of what role such fields may play in the pulsational behavior of these stars. Since the maximum observed field strengths are several hundreds of gauss, they must produce pressures that are comparable to gas pressure in the stellar photospheres. Variability of the fields has been indicated in certain cases, but it is not yet clear whether any periodicity also exists. Although two Cepheids appear not to have measurable fields (Babcock 1958), it is, in any case, an entirely open question as to how the surface field is related to the underlying envelope field. Actually, the detection of a magnetic field in *any* Cepheid needs confirmation (M. A. Smith and M. H. Slovak, private communication). However, a magnetic field has been reported for the spectroscopically similar star γ Cyg in two independent investigations (Severny 1970; Borra and Landstreet 1973). Moreover, if the solar magnetic field is any guide, there may occur protracted periods of very low magnetic-field intensity at the surfaces of all the Cepheids.

Many authors have shown that the masses of Cepheids, as deduced from a combination of their observed pulsational properties and the results of theoretical (nonmagnetic) pulsation calculations, are significantly less than their masses inferred from their observed luminosities combined with a theoretically determined mass-luminosity relation (e.g., Stobie 1974). Although various solutions have been suggested to remove this discrepancy, it is worth asking here what influence an observed factor of evident importance, namely, a magnetic field, has on the derived "pulsational" masses of Cepheids. The present paper addresses this question by making appropriate linearized pulsation calculations of magnetic Cepheid models.

II. ASSUMPTIONS

A magnetic field $\boldsymbol{H}$ has been incorporated in the basic equations of stellar structure and pulsation by employing a rather simple approximation scheme (Trasco 1970; Stothers 1979) in which the field is assumed to be sufficiently small in scale and sufficiently chaotic in arrangement so that the net force it exerts can be considered to be approximately radial. Therefore, the problem reduces formally to a spherically symmetric one. Another simplification is that, because of the high electrical conductivity of the gas, ohmic dissipation of the magnetic field and slippage of the field lines with respect to the moving gas during the course of a pulsation can be assumed to be negligible (except, possibly, very near the surface where most of the gas is neutral). Mathematically, the effect of the magnetic field is to provide a buoyancy force in the equation of motion, the rest of the basic equations (including the thermodynamical relations) remaining the same as in the absence of a magnetic field (see, e.g., Cowling 1953).[1]

For purely heuristic purposes, the equilibrium distribution of the field intensity averaged over a spherical shell is specified to be either uniform with depth or else governed by the relation ν = constant, where ν is the

[1] The neglect of the magnetic field $\boldsymbol{H}$ in the adiabatic processes can be justified very easily. The material in question is an ideal gas of ionized or neutral atomic particles, whose individual magnetic dipole moments μ are of the order of the Bohr magneton. The paramagnetic alignment of the gas in the presence of the applied field $\boldsymbol{H}$ is unimportant in comparison with the random motions of the gas if $\mu H/kT \ll 1$, i.e., if $H \ll 10^4 T$ gauss. This condition is satisfied to many orders of magnitude in Cepheids.

ratio of magnetic pressure ($\langle H^2 \rangle/24\pi$) to thermodynamic pressure. The first distribution typifies a magnetic field that is important only near the stellar surface, while the second distribution allows for a magnetic field that is important in all the pulsating layers. In the atmosphere, however, the mean field intensity is assumed, for simplicity, to be uniform. To compute the radiation flow through the atmosphere, the diffusion approximation is adopted at all layers. Otherwise, the calculations have proceeded as in the case of the nonmagnetic Cepheid models studied earlier by Carson and Stothers (1976). This procedure involved the following steps: (1) computation of the equilibrium structure of the Cepheid envelope; (2) perturbation of this structure in the linear adiabatic approximation in order to derive the periods of radial pulsation; and (3) solution of the linear nonadiabatic pulsation equations in order to determine the stability coefficient. The differential form of the stability coefficient used here is given by equation (C10) of Baker and Kippenhahn (1962), since this form remains unchanged if a magnetic field is present.

For the input physics, standard assumptions are made, since the main purpose here is simply to compare magnetic and nonmagnetic Cepheid models. Thus we adopt standard opacities (Cox and Stewart 1965), no convection in the models, and a (hydrogen, metals) abundance of $(X, Z) = (0.739, 0.021)$.

III. THEORETICAL RESULTS

As typical examples of classical Cepheids, we have computed models in the mass range 4–7 $M_\odot$ with realistic values of effective temperature and luminosity. Specifically, the luminosity was adopted from the "evolutionary" mass-luminosity relation (assuming no mass loss),

$$\log (L/L_\odot) = 4 \log (M/M_\odot) + 0.3 . \qquad (1)$$

For comparative purposes, models were also computed with somewhat lower values of effective temperature, luminosity, and mass. Values of ν running from zero to unity were adopted.

Some results of these calculations are given in Table 1. It is readily seen that, in all cases, the assumption of a uniform mean magnetic field leads to virtually no change in the various periods of radial pulsation if any reasonable value of the photospheric ratio ν_R is adopted. The reason for this is simply that a change of ν_R amounts to little more than a change of the surface boundary conditions, to which the periods of Cepheid models are known to be rather insensitive (e.g., Baker and Kippenhahn 1965).

On the other hand, in the case where ν is spatially invariant, the period of the fundamental mode of pulsation increases noticeably with ν. This is a direct consequence of the large magnetic energy that is contained in the pulsating layers of the star for this case (Chandrasekhar and Limber 1954). If the magnetic field had no other effect on the structure, P_0 would increase in proportion to $(1 + \nu)^{1/2}$ (Stothers 1979)—a proportionality which is approximately obeyed by the actual models. The periods of the higher modes are found to be affected much less by the presence of the magnetic field, and therefore the period ratios P_1/P_0 and P_2/P_0 suffer a mild decline with increasing values of ν. However, the unique relations previously known to exist between P_1/P_0, P_2/P_0, and Q_0 are not affected by the magnetic field. It seems to be a general rule that all the changes of the periods induced by the magnetic field become larger for longer fundamental periods. This can be seen graphically in Figure 1 for a set of short-period models of 5 $M_\odot$ and a set of long-period models of 7 $M_\odot$. Additional models with short periods are presented in Table 2. For other short-period variables, i.e., δ Scuti stars and RR Lyrae stars, supplementary calculations indicate that magnetic fields have little influence on the periods in the former case and an influence similar to that obtained for the classical Cepheids in the latter case.

The situation regarding the theoretical blue edge of the Cepheid instability strip in the H-R diagram is

TABLE 1

THEORETICAL PULSATION CONSTANTS AND PERIOD RATIOS FOR MAGNETIC CEPHEIDS

Parameter	Model 1	2	3	4	5
$M/M_\odot$	5	5	7	7	7
$\log (L/L_\odot)$	3.1	3.7	3.5	3.7	3.7
$\log T_e$	3.81	3.78	3.78	3.72	3.78
P_0 (days)	2.49	10.31	5.44	13.05	8.06
Q_0 (days)	0.0364	0.0435	0.0383	0.0430	0.0402
P_1/P_0	0.748	0.700	0.743	0.707	0.732
P_2/P_0	0.594	0.515	0.578	0.523	0.557
$\Delta P_0/\Delta\nu_R$ (unif.)	0.00	0.15	0.00	0.10	0.02
$\Delta Q_0/\Delta\nu_R$ (unif.)	0.0000	0.0006	0.0000	0.0003	0.0001
$\Delta(P_1/P_0)/\Delta\nu_R$ (unif.)	0.002	0.005	0.002	0.003	0.002
$\Delta(P_2/P_0)/\Delta\nu_R$ (unif.)	0.003	0.002	0.002	0.000	0.002
$\Delta P_0/\Delta\nu$ (nonunif.)	0.55	3.63	1.44	4.37	2.45
$\Delta Q_0/\Delta\nu$ (nonunif.)	0.0080	0.0153	0.0102	0.0144	0.0122
$\Delta(P_1/P_0)/\Delta\nu$ (nonunif.)	−0.045	−0.122	−0.067	−0.125	−0.096
$\Delta(P_2/P_0)/\Delta\nu$ (nonunif.)	−0.072	−0.114	−0.090	−0.125	−0.108

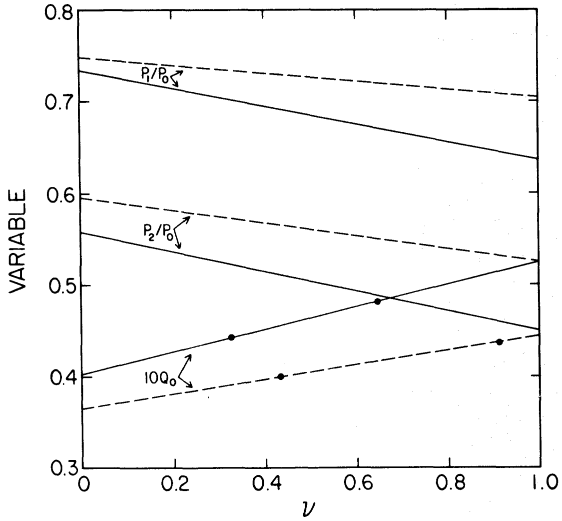

FIG. 1.—Pulsation constant Q_0 and period ratios P_1/P_0 and P_2/P_0 for Cepheid models pervaded by a nonuniform magnetic field with strength parameter ν. Dashed lines refer to $M/M_\odot = 5$, $\log(L/L_\odot) = 3.1$, $\log T_e = 3.81$; solid lines refer to $M/M_\odot = 7$, $\log(L/L_\odot) = 3.7$, $\log T_e = 3.78$. These specifications characterize Cepheids with "evolutionary" masses. The significance of the points is explained in the text.

easily summarized. It is found that both a uniform and a nonuniform mean magnetic field lead to a very similar shift of the blue edge toward lower effective temperatures. For the fundamental mode,

$$\Delta \log T_e \approx -0.01\nu_R\,. \tag{2}$$

The coefficient has a total range of ± 0.01, and is larger in absolute value for cooler blue edges. But with any realistic value of ν_R the shift is unobservably small.

At the surface of the star, as in the deeper layers, the pulsational variation of the magnetic field strength is predicted to follow, approximately, the "adiabatic" relation

$$\frac{\delta\langle H^2\rangle}{\langle H^2\rangle} = \frac{4}{3}\frac{\delta\rho}{\rho}\,. \tag{3}$$

This predicted variation is very large, as full-amplitude calculations of nonmagnetic Cepheid models indicate $\delta\rho/\rho \approx 7$. Although it will be cut down somewhat by virtue of the fact that the surface gas is largely neutral, so that the magnetic lines of force are not as tightly coupled to the moving material as in the deeper layers, we can nevertheless expect the field possibly to become unobservable around the phase of greatest radius expansion. This expectation will follow even if the magnetic field near the surface is distributed in roughly symmetrical flux tubes rather than in the random manner that we have assumed (Kopecky 1963). Unfortunately, the phase at which the observed field strength is least has not yet been accurately determined. Moreover, the observed change of polarity of the field over a cycle (Wood, Weiss, and Jenkner 1977) cannot obviously be explained by a simple pulsational mechanism.

IV. SEMIEMPIRICAL MASSES OF CEPHEIDS WITH NONUNIFORM MAGNETIC FIELDS

a) *Q-Value Cepheid Masses*

The mass of a classical Cepheid can readily be derived from its observed period and radius by applying a theoretically determined pulsation constant

$$Q = P(M/M_\odot)^{1/2}(R/R_\odot)^{-3/2}\,. \tag{4}$$

For well-observed Cepheids the derived "pulsational" masses have turned out to be some 20–40% smaller than the "evolutionary" masses, under the reasonable assumption that the observed period refers to the fundamental mode of radial pulsation. In Figure 1, we have plotted points which represent revised Q-values that are larger by factors of 1.1 and 1.2 than the Q-values computed for the nonmagnetic models. It is evident that the mass discrepancy could be made to vanish if ν were in the range 0.3–0.9.

Burbidge (1956) was the first author to discuss the possibility that an observable period shift could arise from a magnetic field in a Cepheid-like star, in particular RR Lyrae. However, because of the crude state of stellar models two decades ago, his conclusion that the period would decrease is erroneous.

b) *Double-Mode Cepheid Masses*

A number of short-period classical Cepheids display two periods, whose ratio lies in the very narrow range 0.70–0.71. Taking Stobie's (1977) data, we have plotted points representing these stars in Figure 2. Also shown are theoretical predictions for the fundamental mode

TABLE 2

THEORETICAL PERIOD RATIOS FOR SHORT-PERIOD CEPHEIDS WITH NONUNIFORM MAGNETIC FIELDS

			$\nu = 0$			$\nu = 1$		
$M/M_\odot$	LOG $(L/L_\odot)$	LOG T_e	P_0 (day)	P_1/P_0	P_2/P_0	P_0 (day)	P_1/P_0	P_2/P_0
4..................	2.7	3.78	1.69	0.749	0.599	2.05	0.710	0.532
		3.81	1.35	0.752	0.604	1.62	0.719	0.546
5..................	3.1	3.78	3.12	0.745	0.587	3.88	0.690	0.505
		3.81	2.49	0.748	0.594	3.04	0.703	0.522
6..................	3.4	3.78	4.95	0.741	0.575	6.26	0.670	0.482
		3.81	3.93	0.745	0.585	4.87	0.689	0.504

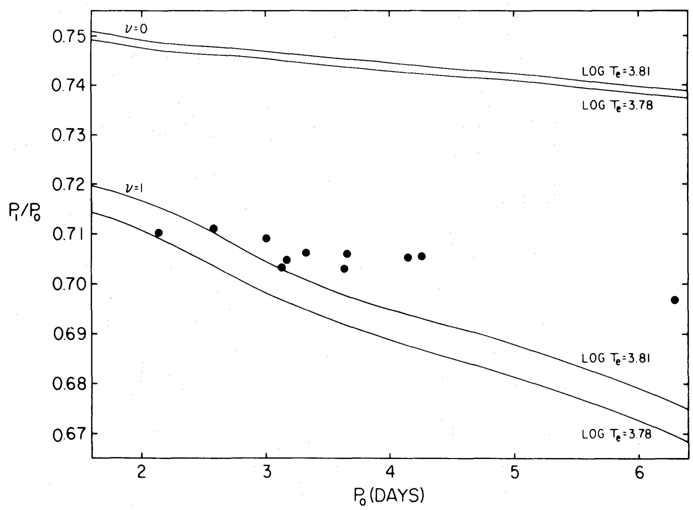

FIG. 2.—Period ratio P_1/P_0 as a function of fundamental period P_0. Lines refer to Cepheid models obeying the "evolutionary" mass-luminosity relation and possessing a nonuniform magnetic field with strength parameter ν. Points represent observed double-mode Cepheids.

and first overtone in Cepheid models obeying the "evolutionary" mass-luminosity relation and employing effective temperatures derived from the observations of beat (double-mode) Cepheids. In the nonmagnetic case, the discrepancy between observation and theory is very large. Notice, however, that the discrepancy can be made to disappear if a magnetic field is assumed to exist with a strength parameter of $\nu = 0.6$–1.0. The inferred masses are then, according to Table 2, quite normal and in the range 4–6 $M_\odot$.

c) *Bump Cepheid Masses*

Nonlinear Cepheid models pulsating at full amplitude (without magnetic fields included) show, in some cases, a secondary bump on their surface-velocity curves. These curves match in detail the observed velocity curves for the same period, provided that the mass is about half the "evolutionary" mass and standard opacities are used in the stellar models. Simon and Schmidt (1976) have suggested that the secondary bump is a direct result of a resonance $P_2/P_0 \approx 0.5$ between the excited fundamental mode and the second overtone. Their semiempirically derived resonance band is defined by $P_2/P_0 = 0.47$–0.53. If this contention is correct, our standard nonmagnetic model for 7 $M_\odot$, which is represented by an "evolutionary" luminosity of $\log(L/L_\odot) = 3.7$ and an effective temperature of $\log T_e = 3.78$ (both characteristic of observed bump Cepheids), would not be expected to show a bump, since $P_2/P_0 = 0.557$. Indeed, nonlinear calculations prove that this model has no bump.

Introduction of a magnetic field, however, lowers the value of P_2/P_0. According to Figure 1, the resonance band of Simon and Schmidt can be fitted by models with "evolutionary" masses if ν is in the range 0.3–0.8. Although the resonance hypothesis as a physical explanation for the bump is very uncertain, it may nevertheless be an approximate phenomenological indicator of when such a bump will appear, and, insofar as this holds true, a magnetic field would obviously help to eliminate the mass discrepancy.

It is instructive to inquire what changes in our results would be incurred by resorting to the Carson (1976) opacities in place of the Cox-Stewart opacities. Using previously published results for nonmagnetic Cepheid models (Carson and Stothers 1976; Vemury and Stothers 1978), we estimate that our present values of ν as inferred from Q_0 and P_1/P_0 would be lowered by about 20%, and that those inferred from P_2/P_0 would be lowered by about 70%.

Although no special importance should be attached to the precise values of ν derived in this preliminary study (for either set of opacities), a value of $\nu \approx 0.8 \pm 0.2$ seems to satisfy all the observational requirements. In particular, for the double-mode Cepheid V367 Sct —a probable member of the open cluster NGC 6649— highly accurate data are available (Stobie 1977) that are entirely consistent with normal evolution if $\nu \approx 0.7$.

V. CONCLUSION

The present investigation of magnetic Cepheid models, although restricted to specific assumptions about the properties of the magnetic field, is nevertheless somewhat more general in its conclusions than may at first sight appear. For one thing, the influence of the magnetic field must probably be regarded as a small perturbation on the radially pulsating Cepheid model, which has been successful in predicting results in *approximate* agreement with observations. Therefore, we believe that our first-order treatment of the magnetic field ought to be adequate even for large values of ν. Second, the approximations entering into the derivation of the average of the field intensity over a spherical shell are restrictive only in that they require an axisymmetric field and one in which

$\langle H_r^2 \rangle = \frac{1}{3}\langle H^2 \rangle$ locally. Although these requirements are clearly best met by a small-scale, well-tangled field, which Tayler's (1974) results on hydromagnetic instabilities may even favor, such a field configuration is not essential; in fact, the *surface* field, in order to be observable, must be largely coherent. Third, our expression for the time variation of the magnetic field strength ($\langle H^2 \rangle \propto \rho^{4/3}$) is probably rather general, as it also holds for a simple homologous displacement of the field lines.

If, then, our mathematical formulation is not particularly restrictive, our specific choice of the mean radial distribution of the field intensity is probably not either. It is only necessary to have a significant total magnetic energy content—in comparison with the total thermodynamic energy content—in the pulsating layers of the star. Since these layers encompass about the same mass of the star for each of the three lowest pulsation modes, the period ratios of these modes are also expected to be qualitatively well described.

How the magnetic field originated is an unanswered question. Nevertheless, a surface magnetic field with a pressure comparable with gas pressure is an observed fact, and we have simply assumed that the ratio of the two kinds of pressure remains more or less the same down to a moderate depth in the star (but not so deep as to affect the overall evolution of the star significantly). Thus the relevant magnetic layer need be only 10^{-3} of the total stellar mass; at the bottom of this layer the field strength would be only $\sim 10^4$ gauss, which is actually less than that observed at the surface of some main-sequence stars.

With this mild assumption, it is possible to explain the observed periods and period ratios of classical Cepheids on the basis of normal "evolutionary" masses. Also explained with normal masses are the secondary bumps on the observed velocity curves, if the Simon-Schmidt resonance hypothesis for the bumps can be applied to the linear results. A further advantage of postulating a strong magnetic field is that such a field would probably help to limit convection (Tayler 1971); it is known from earlier work that convection must be assumed relatively unimportant in Cepheid envelopes in order to predict correctly the observed phase lag at the surface between the light and velocity curves (Castor 1971). Of course, the present hypothesis requires that all Cepheids have strong subsurface magnetic fields. In addition, a large number of approximations have been made in our treatment, particularly regarding the assumed geometry of the magnetic field and the reduction of a complicated three-dimensional problem to a simplified one-dimensional one. Furthermore, the possible changes of the magnetic flux near the surface have not been computed, so that the derived blue edges are also subject to some uncertainty. Nevertheless, in the author's opinion, the present results are astrophysically interesting and suggest that magnetism is an important physical aspect of Cepheids that should be studied much further.

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